

PHYS4150 — PLASMA PHYSICS

LECTURE 18 - PLASMA WAVES

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0.1 *Plasma Oscillations*

What happened if we take away some electrons (say 1%) from some region?

Geometry

$$\mathbf{k} = (k, 0, 0)$$

$$\mathbf{E}_0 = 0$$

$$\delta\mathbf{E} = (\delta E, 0, 0)$$

$$\mathbf{B}_0 = 0$$

$$\delta\mathbf{B} = (0, 0, 0) \quad (\text{electrostatic})$$

$$\mathbf{v}_0 = 0$$

$$\delta v \neq 0 \quad (\text{no flow})$$

$$n_0 \neq 0$$

Poisson equation:

$$\frac{\partial E}{\partial x} = \frac{e}{\epsilon_0} (n_i - n_e)$$

$$\frac{\partial E_0}{\partial x} + \frac{\partial \delta E}{\partial x} = \frac{e}{\epsilon_0} (n_{0i} + \delta n_{0i} - n_{0e} - \delta n_{0e})$$

$$\frac{\partial \delta E}{\partial x} = -\frac{e}{\epsilon_0} \delta n$$

$$ik\delta E = -\frac{e}{\epsilon_0} \delta n$$

$$\delta E = \frac{ie}{\epsilon_0 k} \delta n$$

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Continuity equation:

$$\frac{\partial n_e}{\partial t} + \nabla(n_e v_e) = 0$$

$$\frac{\partial n_{e0}}{\partial t} + \frac{\partial \delta n_e}{\partial t} + \frac{\partial}{\partial x} [(n_{e0} + \delta n_{e0})(v_{e0} + \delta v_{e0})] = 0$$

$$\frac{\partial n_{e0}}{\partial t} + \frac{\partial \delta n_e}{\partial t} + \frac{\partial}{\partial x} [n_{e0} v_{e0} + n_{e0} \delta v_{e0} + \delta n_{e0} v_{e0} + \delta n_{e0} \delta v_{e0}] = 0$$

$$-i\omega \delta n + i k n_0 \delta v = 0$$

$$\delta v = \frac{\omega \delta n}{k n_0}$$

Momentum equation

$$m_e \frac{\partial v}{\partial t} = -eE$$

$$m_e \frac{\partial v_0}{\partial t} + m_e \frac{\partial \delta v}{\partial t} = -eE_0 - e\delta E$$

$$-i\omega \delta v = -\frac{e}{m_e} \delta E$$

$$\delta E = \frac{i\omega \delta v m_e}{e}$$

and hence

$$\frac{i e}{\epsilon_0 k} \delta n = \frac{i \omega m_e}{e} \delta v$$

$$\frac{i e}{\epsilon_0 k} \delta n = \frac{i \omega m_e}{e} \frac{\omega}{k} \frac{1}{n_0} \delta n$$

$$\boxed{\omega^2 = \frac{n_0 e^2}{\epsilon_0 m_e}}$$